Uncertainties and errors in quantitative chemical analysis

When you analyze samples, are you sure that you get correct results?

Lecture Plan

General definitions: mean value, standard deviation, etc.

Types of errors and uncertainty

Method validation

Quality assurance

Mean value

$$\bar{x} = \frac{\sum_{i=1}^{n} x_n}{n}$$

Example: calculate mean value for the following concentrations measurements: 5.32; 5.22; 5.25; 5.35; 5.30; 5.31 (mg/L)

$$\bar{x} = \frac{5.32 + 5.22 + 5.25 + 5.35 + 5.30 + 5.31}{6} = 5.29$$

Mean value in MS Excel

		СРЗНАЧ(С4:С9)	 ✓ f_{st} = 	• : >	C10
	D	С	В	А	
СРЗНАЧ(диапазон)	value	n of mean	Calculatio		1
					2
Or		C, mg/L	N		3
		5,32	1		4
AVERAGE(range)		5,22	2		5
		5,25	3		6
		5,35	4		7
-		5,30	5		8
-		5,31	6		9
-		5,29	Mean		10
					11

Standard deviation

$$S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Example: calculate standard deviation value for the following concentrations measurements: 5.32; 5.22; 5.25; 5.35; 5.30; 5.31 (mg/L)

$$S = \sqrt{\frac{(5.32 - 5.29)^2 + (5.22 - 5.29)^2 + (5.25 - 5.29)^2 + \dots}{5}} = ?$$

Standard deviation inMS Excel

С11 · : × ✓ ƒx =стандотклон(с4:с9)							
	А	В	C	D			
1		Calculatio	Calculation of mean value				
2							
3		Ν	C, mg/L				
4		1	5,32				
5		2	5,22				
6		3	5,25				
7		4	5,35				
8		5	5,30				
9		6	5,31				
10		Mean	5,29				
11		S	0,048				
10							

СТАНДОТКЛОН (диапазон)

or

STDEV(range)



Gaussian distribution



Confidence interval

Confidence interval = $\bar{x} \pm \frac{t \times s}{\sqrt{n}}$

Where: t - Student's t coefficient; n - number of measurements

Example: calculate **confidence interval** (95%) value for the following concentrations measurements: 5.32; 5.22; 5.25; 5.35; 5.30; 5.31 (mg/L)

Student's t coefficients

TABLE 4-2 Values of Student's t

Degrees	Confidence level (%)								
of freedom	50	90	95	98	99	99.5	99.9		
1	1.000	6.314	12.706	31.821	63.656	127.321	636.578		
2	0.816	2.920	4.303	6.965	9.925	14.089	31.598		
3	0.765	2.353	3.182	4.541	5.841	7.453	12.924		
4	0.741	2.132	2 776	3.747	4.604	5.598	8.610		
5	0.727	2.015	2.571	3.365	4.032	4.773	6.869		
6	0.718	1.943	2.447	3.143	3.707	4.317	5.959		
7	0.711	1.895	2.365	2.998	3.500	4.029	5.408		
8	0.706	1.860	2.306	2.896	3.355	3.832	5.041		
9	0.703	1.833	2.262	2.821	3.250	3.690	4.781		
10	0.700	1.812	2.228	2.764	3.169	3.581	4.587		
15	0.691	1.753	2.131	2.602	2.947	3.252	4.073		
20	0.687	1.725	2.086	2.528	2.845	3.153	3.850		
25	0.684	1.708	2.060	2.485	2.787	3.078	3.725		
30	0.683	1.697	2.042	2.457	2.750	3.030	3.646		
40	0.681	1.684	2.021	2.423	2.704	2.971	3.551		
60	0.679	1.671	2.000	2.390	2.660	2.915	3.460		
120	0.677	1.658	1.980	2.358	2.617	2.860	3.373		
∞	0.674	1.645	1.960	2.326	2.576	2.807	3.291		

Confidence interval in Excel

E13 ▼ : × ✓ <i>f</i> _x =C12*C11/КОРЕНЬ(В9)								
	A	В	С		E	F		
1		Calculatio	Calculation of confidence interval					
2								
3		N	C, mg/L					
4		1	5,32					
5		2	5,22					
6		3	5,25					
7		4	5,35					
8		5	5,30					
9		6	5,31					
10		Mean	5,29					
11		S	0 <mark>,</mark> 048					
12		t	2,57					
13		conf int	5,292	±	0,050	mg/L		

50% and 90% confidence intervals



Types of errors

Systematic – due to poor design of experiment, equipment or method

Random – due to any uncontrolled variables

Accuracy and precision

Accuracy shows how close results are to the "real" value

Precision shows how reproducible your results are

Reproducibility – analyses are done several times by different people, in different laboratories, using different equipment, at different conditions, etc.



Accuracy and precision

Accuracy and precision



Accurate & imprecise

Inaccurate & precise

Inaccurate & imprecise



Vodka contains 380 mg/L of methanol You got the following results of analysis (mg/L): 350 370 360

Calculate accuracy and precision. How would you characterize them (high or low)?

Why do you need accuracy and precision?

Accuracy:

Imagine that you drug must contain certain concentration. A 5% higher concentration will kill you (>lethal dose), but 5%lower will not work as it should (drugs against cancer).

If you perform disease screening, differences in urine and blood concentrations of certain metabolites between sick and healthy patients can be very low. You do not want to be false diagnosed a decease.

Precision:

If the method is imprecise, you will have to analyze many replicates to get accurate results. That will require a lot of extra time and money

Uncertainty

Shows the maximum error of chemical analysis

A "sum" of all systematic and random errors during analysis

Example: concentration of quercetin in *Artemisia* plants is 2.31 ±0.05 mg/kg

Absolute uncertainty is 0.05 mg/kgRelative uncertainty =(0.05 mg/kg) / (2.31 mg/kg) = 0.022Or in %:0.022 *100% = 2.2%

Summary of rules

TABLE 3-1 Summary of rules for propagation of uncertainty

Function	Uncertainty	Function ^{<i>a</i>}	Uncertainty ^b
$y = x_1 + x_2$	$e_y = \sqrt{e_{x_1}^2 + e_{x_2}^2}$	$y = x^a$	$\% e_y = a\% e_x$
$y = x_1 - x_2$	$e_y = \sqrt{e_{x_1}^2 + e_{x_2}^2}$	$y = \log x$	$e_y = \frac{1}{\ln 10} \frac{e_x}{x} \approx 0.43429\frac{e_x}{x}$
$y = x_1 \cdot x_2$	$\% e_y = \sqrt{\% e_{x_1}^2 + \% e_{x_2}^2}$	$y = \ln x$	$e_y = \frac{e_x}{x}$
$y = \frac{x_1}{x_2}$	$\% e_y = \sqrt{\% e_{x_1}^2 + \% e_{x_2}^2}$	$y = 10^x$	$\frac{e_y}{y} = (\ln 10)e_x \approx 2.302 \ 6 \ e_x$
		$y = e^x$	$\frac{e_y}{y} = e_x$

a. x represents a variable and a represents a constant that has no uncertainty. b. e_x/x is the relative error in x and $\Re e_x$ is $100 \times e_x/x$.

Uncertainty: example

Find the total amount of impurities in gold if concentration of:

- 1) Silver is 5.21 ± 0.05%;
- Nickel 1.11 ± 0.04%;
- 3) Titanium 0.22 ± 0.02%

The total concentration of impurities in gold is 5.21+1.11+0.22 = 6.54% What is uncertainty of this value?

$$e_{total} = \sqrt{\sum_{i=1}^{n} e_i^2} = \sqrt{(0.05)^2 + (0.04)^2 + (0.02)^2} = 0.07$$

Relative uncertainty = (0.07% / 6.54%) × 100% = 1.0%

Calculations in MSExcel

D8	• ÷ ×	✓ ƒ _x =КОРЕНЬ(D5^2+D6^2+D7)	7^2)				
	А	В	С	D			
1		Calculation of the	results				
2		Impurities in gold s	Impurities in gold sample				
3							
4			С	е			
5		Silver	5,21	0,05			
6		Nickel	1,11	0,04			
7		Titanium	0,22	0,02			
8		Total	6,54	0,07			
9		Relative e, %		1,0			
10	۱						

S

Uncertainty: example 2

Find the concentration of solution if 10.4 ± 0.1 mg of BaCl₂ (purity 99.95 ± 0.05%) were weighted using analytical balances and dissolved in water to the final volume 50.00 ± 0.02 mL. Find the uncertainty of the concentration of BaCl₂

The concentration of BaCl₂ in solution can be found using:

$$C = \frac{m \times Purity}{V \times 100\%} = \frac{10.4 \ mg \times 99.95 \ \%}{50.00 \ mL \times 100 \ \%} = 0.207896 \ \textbf{?} =$$

What is the uncertainty of this value?

$$rel e_{total} = \sqrt{\sum_{i=1}^{n} rel e_i^2}$$

MS Excel calculations

E8	• :	× √ ƒ _ж =КОРЕНЬ(Е5^2+	E6^2+E7^2)			
	A	В	С	D	E	
1		Calculation of the	Calculation of the results			
2		Concentration of E				
3						
4			Value	е	rel e (%)	
5		Mass, mg	10,4	0,1	0,96	>
6		Purity, %	99,95	0,05	0,05	
7		Volume, mL	50,00	0,02	0,04	
8	Concentration, mg/mL		0,208	0,002	0,96	
9						

Task

Calibrated gas sampling bulb (V = 250 ± 2 mL) was spiked with 10.0 ± 0.2 µL of benzene solution in methanol (C = 15.0 ± 0.2 ng/µL). Calculate the concentration of benzene in the bulb and it's uncertainty

$$C_{2} = \frac{C_{1}V_{1}}{V_{2}} = \frac{15\frac{ng}{\mu L} \times 10\ \mu L}{250\ mL} = \frac{150\ ng}{250\ mL} = 0.600\ \frac{ng}{mL}$$
$$\% e_{C_{2}} = \sqrt{\% e_{C_{1}}^{2} + \% e_{V_{1}}^{2} + \% e_{V_{2}}^{2}}$$
$$\% e_{C_{1}} = \frac{0.2}{15.0} \times 100\% = 1.33\%$$
$$\% e_{V_{1}} = \frac{0.2}{10.0} \times 100\% = 2.0\%$$
$$\% e_{V_{2}} = \frac{2}{250} \times 100\% = 0.80\%$$

Solution (continued)

$$\% e_{C_2} = \sqrt{6.409\%^2} = 2.53\%$$

$$e_{C_2} = \frac{C_2 \times \% e_{C_2}}{100\%} = \frac{0.600 \ \frac{ng}{mL} \times 2.53\%}{100\%} = 0.01518 \frac{ng}{mL}$$

Answer: $C_2 = 0.600 \pm 0.015$

Exercise

10.0 g of soil containing N-nitrosodimethylamine (NDMA) was extracted by two portions of acetone (25 mL each). Extracts were combined to the final volume 42±2 mL and evaporated to the final volume 1.00±0.02 mL. Analysis of the evaporated extract by GC-MS showed NDMA concentration 27±1 ng/mL. Calculate NDMA concentration in soil sample and its uncertainty.

Solution

From the available data, we can calculate the total mass of analyte in concentrated extract:

$$m = V \times C = 27 \frac{ng}{mL} \times 1.00 \ mL = 27 \ ng$$

If no analyte was lost during evaporation, this analyte mass was also present in the extract before evaporation. Extraction recovery was not provided. Some portion of solvent remained in soil during filtration. But probably, the mass of NDMA in this part is negligible and recovery is 100%.

$$C = \frac{m_{NDMA}}{m_{soil}} = \frac{27 \ ng}{10.0 \ g} = 2.7 \ \frac{ng}{g}$$

General formula

$$m = V \times C_{ex} = 27 \frac{ng}{mL} \times 1.00 mL = 27 ng$$

$$C_s = \frac{m_{NDMA}}{m_{soil}} = \frac{27 \ ng}{10.0 \ g} = 2.7 \ \frac{ng}{g}$$

$$C_{s} = \frac{V \times C_{ex}}{m_{soil}} = \frac{27 \ \frac{ng}{mL} \times 1.00 \ mL}{10.0 \ g} = 2.7 \ \frac{ng}{g}$$

Uncertainty

$$\% e_{C_s} = \sqrt{\% e_{C_{ex}}^2 + \% e_V^2 + \% e_m^2}$$

$$\% e_{C_{ex}} = \frac{1}{27} \times 100\% = 3.7\%$$

$$\% e_V = \frac{0.02}{1} \times 100\% = 2\%$$

$$\% e_m = \frac{0.1}{10.0} \times 100\% = 1\%$$

Uncertainty

$$\% e_{C_s} = \sqrt{3.7^2 + 2^2 + 1^2} = 4.3\%$$

$$e_{C_s} = 2.7 \frac{ng}{g} \times \frac{4.3\%}{100\%} = 0.12 \frac{ng}{g}$$

Answer: NDMA concentration in analyzed soil sample is 2.70±0.12 ng/g

Exercise

Brandy sample was analyzed for the concentration of iso-butanol using standard addition method. Standard additions were 0.50; 1.00; 2.00; 5.00; 10.0 and 20.0 mg/L. Peak areas of iso-butanol peaks were 6.23; 7.44; 9.46; 15.5; 25.5 µ 45.3 arbitrary units. Calculate the concentration of iso-butanol in the analyzed sample and it's uncertainty.

Significant figures: Real Rule

 $\frac{0.002364 (\pm 0.00003)}{0.02500 (\pm 0.00005)} = 0.0946 (\pm 0.0002)$

 $\frac{0.002664 (\pm 0.00003)}{0.02500 (\pm 0.00005)} = 0.1066 (\pm 0.0002)$

 $\frac{0.821 (\pm 0.002)}{0.803 (\pm 0.002)} = 1.022 (\pm 0.004)$

Rule: when first number in the answer is 1 or 9, there can be one extra or one lacking significant figure, respectively.

Significance of difference

Case 1: you analyzed certified reference material (CRM) having known concentration of analyte. Are your results significantly different from the certified value?

Answer:

1) analyze sample several times;

2) calculate 95% confidence interval for replicate measurements;3) if certified concentration does not lie in your confidence interval, the results are "different".

Example: CRM of coal contains 3.19% of sulfur. You analyzed sample 4 times and got the following concentrations of sulfur: 3.29; 3.22; 3.30 and 3.23%. Does your answer agree with the known answer?

Solution in MS Excel

Agreemen	t of data							
CRM of sulfur by new method								
N	C, %							
1	3.29				Con			
2	3.22				1) C			
3	3.30				con			
4	3.23				2)Re			
Mean	3.26				sign			
S	0.041							
t	3.18							
conf int	3.260	±	0.065	%				
CRM	3.19		0.070	Mean-CRM				

Conclusion:

 CRM does not lie in confidence interval;
 Results are significantly different

Significance of difference

Case 2: comparing replicate measurements.

You and your colleague analyzed sample using 2 different methods. Are your results significantly different?

$$t_{calculated} = \frac{|\overline{x_1} - \overline{x_2}|}{S_{pooled}} \sqrt{\frac{n_1 \times n_2}{n_1 + n_2}}$$

$$S_{pooled} = \sqrt{\frac{S_1^2 (n_1 - 1) + S_2^2 (n_2 - 1)}{n_1 + n_2 - 2}}$$

if $t_{calculated} > t_{table}$, results are different

Example in MS Excel

$\times \checkmark f_x =$	КОРЕНЬ(((С15^2)*(С16-	1)+E15^2*(E16-1))	/(C16+E16-2))
В	С	D	E
	Difference be	etween 2 s	ets of measurements
	Mass of nitro	gen isolate	ed from air
	From air (g)		From chemical
			decomposition (g)
1	2.31017		2.30143
2	2.30986		2.29890
3	2.31010		2.29816
4	2.31001		2.30182
5	2.31024		2.29869
6	2.31010		2.29940
7	2.31028		2.29849
8			2.29889

Conclusion: 1)t_{calc} >t_{table}; 2)Two sets of measurements are significantly different

Grubbs test for Outliers

You analyzed your sample several times and need to find an outlier.

$$G_{calculated} = \frac{|questionable value - \bar{x}|}{S}$$

Data points having $G_{calculated} > G_{table}$ are outliers.

Homework: Understand example on Page 83 (Harris book), section 4-6

TABLE 4-5Cfor rejection o	of outlier
Number of observations	<i>G</i> (95% confidence)
4	1.463
5	1.672
6	1.822
7	1.938
8	2.032
9	2.110
10	2.176
11	2.234
12	2.285
15	2.409
20	2.557

Least squares method

Used to get linear calibration curve (Signal = f (Concentration): S=aC + b



Linear range of calibration

The linear range of an analytical method is the analyte concentration range over which response is proportional to concentration

Dynamic range - the concentration range over which there is a measurable response to analyte, even if the response is not linear



Least squares in MS Excel

G3	• :	$\times \checkmark f_x$	{=ЛИНЕЙН(С3:С7,	В3:В7,ИСТИНА,И	1СТИНА)}				
	A	В	С	D	Е	F	G	Н	
1									
2		C, ug/L	S	SD			Slope	Intercept	
3		30	356.6	13.3		Parameter	2.797	272	
4		50	390.1	35		SD	0.046	12	
5		100	572.6	37.2		R2	0.9992	19	Sy
6		300	1119.8	63.8					
7		500	1663.9	186.7					

Use "ЛИНЕЙН" or "LINEST" functions:

- 1) Select 3x3 cell array;
- Enter the formula, select Y and X ranges (see above);
- 3) Last 2 variables must have "TRUE" and "TRUE" values;
- 4) Hold "SHIFT+CTRL" buttons, press "ENTER" button.

Linear plot equation: $S = (2.797 \pm 0.046) \times C + (272 \pm 12)$

Calibration plot with error bars



Error bars in MS Excel 2013



Error bars	s in I	MS	Exce	el 20	13	
ПАРАМЕТРЫ ПРЕДЕЛА ПОГРЕШНОСТЕЙ ▼ ⑦						
ВЕРТИКАЛЬНЫЙ ПРЕДЕЛ ПОГРЕШНОСТИ Направление						
О <u>М</u> инус						
Т ○ <u>П</u> люс						
Стиль края						
<u>Б</u> ез точки			C, ug/L	S	SD	
® <u>Т</u> очка			30	356.6	13.3	
Величина погрешности:			50	390.1	35	
о фиксированное 0.1			100	572.6	37.2	
относительное 5.0 %			300	1119.8	63.8	
стандартное 1.0			500	1663.9	186.7	_
стандартная погре <u>ш</u> ность						/ = -
пользовательская <u>Укажите значение</u>	- Sele	ct rang	e			

Propagation of uncertainty

$$S_x = \frac{S_y}{|a|} \sqrt{\frac{1}{k} + \frac{1}{n} + \frac{(y - \bar{y})^2}{a^2 \sum (x_i - \bar{x})^2}}$$

k - number of replicate measurements of unknown sample;

n – number of data points in calibration plot;

a – slope;

 \bar{y} - mean value of Y for the points of calibration plot;

 \bar{x} - mean value of X for the points of calibration plot

Uncertainty propagation in Excel

F13	F13 • : × \checkmark f_x =(B13-\$C\$8)^2/\$H\$3^2/\$B\$9										
	А	В	С	D	E	F	G	Н	L	J	
1											
2		C, ug/L	S	SD				Slope	Intercept		
3	1	30	356,6	13,3			Parameter	2,797	272		
4	3	50	390,1	35			SD	0,046	12		
5	4	100	572,6	37,2			R2	0,9992	19	Sy	
6	6	300	1119,8	63,8							
7		500	1663,9	186,7							
8	Mean	196	820,6								
9	DEVSQ	161320									
10											
11	Real sample analyses										
12		S mean	С	K	n	3	SR	Sx	t	conf. int	
13		800	188,6342	2	5	0,0003363	0,836861	5,541378	6,313752	34,98688	
14	,	800	188,6342	5	5	0,0003363	0,632721	4,189642	2,131847	8,931676	
15		800	188,6342	10	5	0,0003363	0,548029	3,628845	1,833113	6,652082	
16											
17		400	45,60892	2	5	0,1402026	0,916626	6,069549	6,313752	38,32163	
18		400	45,60892	5	5	0,1402026	0,734985	4,866792	2,131847	10,37525	
19		400	45,60892	10	5	0,1402026	0,663478	4,393299	1,833113	8,053413	
20											

Method validation

Specificity (selectivity)

Linearity

Accuracy

Precision

Concentrations range

Limits of detection and quantification

Method accuracy check

Analyze certified reference material or spiked sample

Compare your results with certified value or concentration spiked

Check if the differences are significant (see slides above)

Spike recovery

$$\% recovery = \frac{C_{spiked \ sample} - C_{unspiked \ sample}}{C_{added}} \times 100\%$$

We established benzene concentration in water sample 10.0 μ g/L. A benzene spike of 5.0 μ g/L was added to a replicate portion of sample. Analysis of the spiked sample gave a concentration of 14.6 μ g/L. Find the percent recovery of the spike

% recovery =
$$\frac{14.6 \,\mu\text{g/L} - 10.0 \,\mu\text{g/L}}{5.0 \,\mu\text{g/L}} \times 100\% = 92\%$$

Acceptable recovery depends on the method requirements. For environmental samples, recovery 50-120% is generally acceptable



Soil sample (10.0 g) having DDT concentration 5.0 μ g/kg was spiked with 10.0 μ L of DDT solution in hexane with concentration 100 ng/ μ L. Analysis of the spiked sample gave the DDT concentration 17 μ g/kg. Calculate spike recovery

Method precision check

Instrument: single aliquot, day, instrument, analysts

Intra-assay: multiple aliquots, single day, instrument, analyst

Intermediate: multiple people, days, instruments, analysts in the same lab

Interlaboratory: multiple people, days, instruments, analysts and laboratories

Limit of quantification (LOQ)

The minimum concentration of analyte that can be analyzed with a desired error (for example, <10%)

Typically, predicted using calibration curve and standard deviation values of calibration solutions



Limit of detection (LOD)

Limit of detection is the smallest quantity of analyte that is "significantly different" from the blank

For chromatography, if separation is very efficient and baseline is stable, it may correspond to concentration that gives peak having 3:1 signal to noise ratio

Most popular method:

1) Analyze concentration that is close to the smallest concentration on calibration plot in 7 replicates;

2)Calculate standard deviation (SD) of concentration determined using calibration plot;

3)Multiply SD by t-coefficient (for 99% confidence and 6 degrees of freedom) – 2.896

Task

Sample with NDMA (N-nitrosodimethylamine) concentration in water close to the limit of quantification was analyzed in 7 replicates. The results are (μ g/L):

3.5; 3.1; 4.0; 3.7; 4.2; 3.6; 3.2

Calculate detection limit for the method of NDMA determination in water

Quality assurance is what we (e.g., analytical chemists) do to provide sufficient accuracy and precision of results of chemical analyses

There is no point in spending extra money to obtain a more accurate or more precise answer if it is not necessary

Laboratory accreditation

МЕЖГОСУДАРСТВЕННЫЙ Стандарт

17025-2009

ГОСТ ИСО/МЭК

ОБЩИЕ ТРЕБОВАНИЯ К КОМПЕТЕНТНОСТИ

ИСПЫТАТЕЛЬНЫХ И КАЛИБРОВОЧНЫХ ЛАБОРАТОРИЙ

ISO/IEC 17025:2005

General requirements for the competence

of testing and calibration laboratories

(IDT)

Quality assurance methods

Analysis of blank (not containing analyte) samples

Spike recovery

Control charts

Analyses of performance test samples

Monitoring of equipment and all materials

Regular interview and training of laboratory personnel

These are responsibilities of a Laboratory Manager





